Andrea M. Guillaume and Lisa Kirtman

Learning Lessons about Lessons: Memories of Mathematics Instruction

"My ninth-grade algebra teacher changed my attitude toward math. He was a dynamic teacher who knew how to make math understandable and relate to real life. I loved going to his class, and soon found myself doing well in math again. It took a while, but gradually I regained my love of math."

This quote, written by a future teacher, illustrates the powerful effect that early life experiences can have on teachers of mathematics. People who become mathematics teachers have decades of classroom experience that shape their visions of mathematics and their teaching practices long before they ever step to the front of the classroom. Your own experiences with mathematics will influence your reading of this article, so we invite you to take a moment to ponder your earlier (preschool to college) experiences with mathematics by considering some or all of these reflective questions:

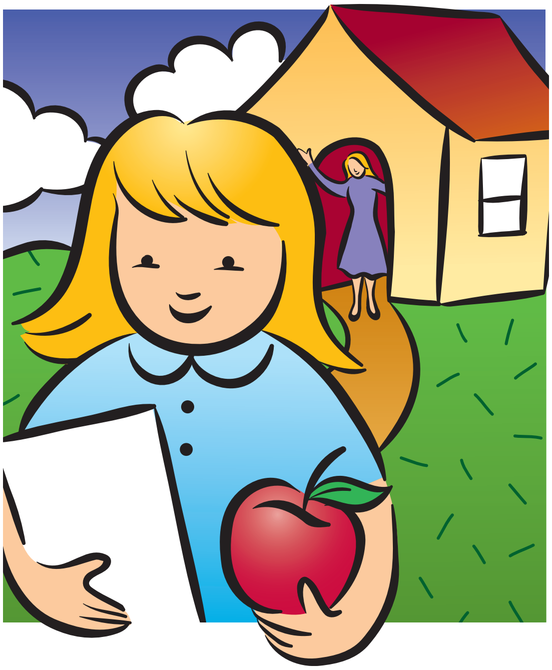
- Growing up, what kinds of mathematical experiences did you engage in with your family or others outside school?
- Who were the powerful mathematics teachers in your life?
- Were certain topics of mathematics study prevalent in your experience?
- How much did school mathematics emphasize conceptual understanding? Computational proficiency? Problem solving?
- Were there "typical" behaviors or strategies that teachers employed during mathematics instruction?
- Which teacher behaviors supported your growth in mathematics?

Answers to reflective questions such as these hold important implications for policy and practice, as the professional literature indicates. Research suggests that teachers' beliefs and practices are shaped by years of experience with mathematics as students in school and beyond (Ball, Lubienski, and Mewborn 2001). Given prevalent instructional practices in America, many students (Kloosterman, Raymond, and Emenaker 1996; Sylvester 1980) and teachers (Wilkins 2000; Wilson 2003) view mathematics narrowly as the study of numbers and algorithms and strive for computational proficiency as the dominant perceived goal. Furthermore, when teachers have negative experiences as students of mathematics, these experiences can impede their use of mathematics in the classroom (Gabriele and Joram 1998; Grant 1996; Johnston and Whitenack 1992; Riedesel and Schwartz 1994). For example, teachers who as students experienced mathematics solely as the application of algorithms are less likely to teach mathematics as a conceptual system. It has been argued that limits in confidence and comfort also limit practice (McDuffie 2004). Teachers tend to emphasize subjects that they are more comfortable teaching, and they tend to teach subjects they like with more enthusiasm than those with which they have had negative experiences. Fortunately, recent mathematics reforms, such as those advocated by the National Council of Teachers of Mathematics (2000), have focused on re-examining the "technical model of teaching" (McDuffie 2004) and have attempted to create reflective practices and to allow students to develop deeper and richer conceptualization of mathematics.

As teacher educators, we (the authors) are vitally interested in drawing from prospective

Andrea Guillaume, aguillaume@fullerton.edu, is a professor in the Department of Elementary and Bilingual Education at California State University in Fullerton, California. She is interested in helping new teachers assist their students in developing conceptual understanding and problem-solving competence. Lisa Kirtman, lkirtman@fullerton.edu, is an assistant professor in the Department of Elementary and Bilingual Education at California State University. She is interested in new teacher development and support in the area of mathematics education.

Edited by **Julie Cwikla,** *julie_cwikla@yahoo.com, University of Southern Mississippi Gulf Coast, Long Beach, MS 39560, and* **Armando Martinez-Cruz,** *amartinez-cruz@fullerton.edu, California State University—Fullerton, Fullerton, CA 92834. Readers are encouraged to send manuscripts appropriate for this section to "Research, Reflection, Practice," NCTM, 1906 Association Dr., Reston, VA 20191-1502; or send electronic submissions to tcm@nctm.org.*



teachers' existing beliefs and notions as we guide them in becoming effective teachers of mathematics who encourage rich and deep notions of mathematics for students. Therefore, we conducted an investigation of the mathematical experiences that may have influenced future elementary teachers' current views of mathematics and mathematics instruction. Through study of mathematics autobiographies written by 144 preservice teachers, we found lessons that have implications for teacher educators and classroom teachers. This article invites the reader to reflect on past experiences with mathematics and shares the lessons we learned from preservice teacher participants' mathematics histories. It also explores some questions that findings raise for classroom teachers in reflecting on the effects of mathematics instruction.

Mathematics Autobiographies

Before engaging in any methods coursework or field experiences, participants wrote their personal histories, or mathematics autobiographies. We gave them the following prompt:

> Please compose your autobiography as a mathematician, that is, as a learner and knower of mathematics. Go as far back in your own life as you can recall, and include information that you see as pertinent from both in-school and out-of-school experiences. You may wish to consider people and experiences that had strong effects on you, whether those effects were positive or negative.

Participants also represented their mathematical histories by converting them to line graphs, or timelines, that captured their self-rating of their mathematical performance and their attitude toward mathematics. **Figure 1** shows a sample timeline. Participants discussed their autobiographies and timelines in class. The productivity of those discussions suggests that individual interviews with participants might enrich future studies.

Results and Discussion

What do students learn while they learn mathematics? Of the many kinds of lessons we found through analysis of the themes in participants' autobiographies, two are particularly relevant here: lessons about mathematics, and lessons about teaching and learning. In both of these types of lessons, participants reported "learning" the intended curriculum, as well as some powerful lessons that were perhaps unintended.

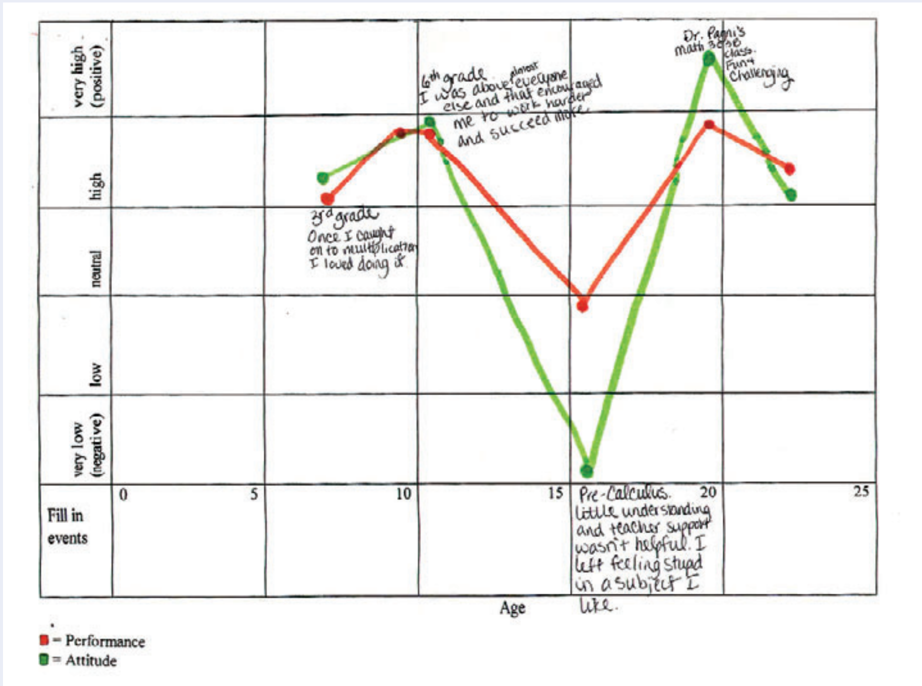
Lessons about mathematics

Through their prior experiences, participants in our study learned potent lessons about the nature of mathematics and what it means to know mathematics. First, they learned that mathematics is a school subject. Unlike reading, writing, or social studies, for which many people report regular out-of-school use of the content or its related processes (Grant 1996), mathematics was seen by most of the participants as something that occurred solely or primarily in school. Of the 144 participants, only 60 (42 percent) discussed a memory of some type of home connection to mathematics. Family or home connections with mathematics were most prevalent during the early years, beginning in preschool with counting experiences, and they faded dramatically by high school. Furthermore, 65 percent of the reported family memories were of activities such as flash cards, workbooks, and tutoring or help with homework. Thus, the majority of the home connections with mathematics stemmed from school so that most participants (123 of 144; 85 percent) reported either no family experiences with mathematics or solely school-related experiences with mathematics at home.

Second, participants learned that mathematics is primarily procedural and algorithmic. As consis-

Figure 1

A sample timeline representation of a participant's mathematics autobiography



tent with past research—and despite the fact that most participants attended K–12 schooling during the time of recent reforms—participants recalled the pursuit of procedural proficiency far more often than they recalled conceptual understanding or problem solving as instructional goals. For example, 84 percent of participants' essays discussed procedural competence as a focus of instruction at the elementary level. Just 23 percent of the essays recalled conceptual understanding as a focus, and a mere 14 percent of the essays mentioned problem solving as a focus of instruction at the elementary level. Memorization of multiplication facts was a nearly universal elementary school memory:

> The earliest elementary school memories of math that I have are from Mrs. W's third-grade class. I can still to this day remember the smell of the chocolate sticker she placed at the top of my "twelve's times tables" one-minute quiz. I was very proud of myself that I had actually learned my twelve's times tables, and every other times table up to that point.

Also in terms of procedural proficiency, participants learned the algorithms for basic operations, often absent from conceptual understanding:

> I remember learning how to add two-digit numbers and feeling a bit inadequate when I had trouble remembering which of the two numbers to carry. Everybody else seemed to get it and I was embarrassed to keep asking the same question.

Problem solving as a goal was discussed with low frequency at all levels: elementary, junior high, high school, and college.

Lessons about teaching and learning

Experiences as mathematics students taught our participants lessons about standard practice and about the power of excellent teachers.

Standard practice. Participants in our study remembered experiencing instruction that is similar to that captured in national (the NAEP, as reported in Silver and Kenney 2000) and international (Trends in Mathematics and Science Study; National Center for Education Statistics 2003) studies of mathematics instruction in America.

Namely, participants told of algorithm-based instruction focused largely on drill and memorization. Text-based, direct instruction lessons seemed prevalent; 65 percent of the essays shared memories of typical textbook-driven instruction and 59 percent discussed direct instruction as the prevalent instructional model. Conversely, only 17 percent of the essays recalled solving real-life-based problems, and only 8 percent included mention of using technology of any sort in mathematics. For example, one participant stated, "Most of my mathematical education in elementary school consisted of traditional bookwork, dittoes, and tests. I do not remember many hands-on experiences." The quote in **figure 2** hints at similar experiences for another participant. Although **figure 2**'s author disliked traditional instruction, participants attached different evaluations to these standard classroom practices. Instructional practices such as lecture, students solving exercises at the board, competitive games, and worksheets were sometimes valued and sometimes resented, depending on factors such as the participants' success and preferences and their perception of the skill of the teacher.

As a result of the general patterns of mathematics instruction in teachers' memories up and down the grades, it may indeed be the case that "most adults graduate from school never having experienced any of the power, elegance, and beauty of the subject" (Ball, Lubienski, and Mewborn 2001, p. 435).

The power of excellent teachers. Despite the traditional mathematics instruction participants in our study received, they rated their own performance in and attitude toward mathematics quite positively, and they encountered many teachers who taught them long-lasting lessons about just how powerful a teacher can be. Of our 144 participants, 98 (68 percent) discussed the power that one teacher (or more teachers) wielded to shape participants' successes and failures with mathematics. Of these 98 participants, 73 (75 percent) argued that one teacher changed their view of mathematics in ways that had long-lasting effects on their views of mathematics, for better and worse. Participants' stories included teachers from all levels, elementary through college, some with negative results:

- I remember in second grade my teacher would yell at us students when we did not understand math. . . . I think it was from this point on I dreaded math.
- I can feel the sweat coming out on my face and that hot feeling jetting up from my feet up through my stomach and up my neck only to sting my checks, and at the same time my heart is pounding so loud I fear that everyone can hear it. [My junior high math teacher] comes toward me and takes the chalk out of my hand and says, "You are just a very stupid girl." This moment affected me through my high school and college years. . . . I went to college, but I could not finish and receive a degree because I could not bear the thought of another math class.
- [My college instructor's] insensitivity and the trauma of failing the course produced a lifelong fear and dislike of anything that had to do with math.

Fortunately, participants also worked with powerful teachers who affected them in positive ways:

- My sixth-grade teacher was the one who inspired me to see math differently.
- She was my teacher for a whole year and I liked her very much; I firmly believe that because of her I learned to like math.
- The impact of my Math 200B teacher will stay with me forever. For the first time in my life I truly enjoyed attending math class and even received an A. This was a major turning point for me.

Future teachers often mediated powerful teachers' effects. The harm done by one teacher might

Figure 2

One teacher's memories of traditional instructional practices

Computation! That's what it was all about when it came to math in my elementary school. The teacher began his math lesson in the same predictable manner every morning: "Please turn to page [56] and watch as I work out this sample problem on the board." After working out two or three problems and briefly explaining the steps, he would tell us to "complete problems 1–40 on page [57 and 58] for tomorrow." That was it! No real-world exposure, no practical application. The following day we would go on to another concept. All this culminated in a weekly, Friday test. . . . Sure, we would understand how to compute and solve the problem, but it all ended there. "Why do we need to know this?" and "How am I ever going to use this in life?" were questions that continuously came up by my classmates. The math concepts never "stepped out" of the book into real life.

be undone by another teacher perceived as positive, and vice versa. The "peak and valley" nature of 81 percent of participants' timelines (recall the sample in **fig. 1**) indicates that participants' attitude (and performance) self-ratings were plastic and subject to future modification, often by teachers. Based on participants' indelible memories it is clear: Mathematics teachers can make a difference—often a life-long difference—to their students.

Notions of good practice. Through their years observing mathematics instruction, participants entered their credential program with clear notions about what it means to teach well. Six themes arise in participants' stories, and we were heartened by our participants' lived experience of practices that are consistent with many current recommendations for effective mathematics instruction.

1. Good teachers believe in their students and convey that conviction.

Some powerful teachers taught participants lessons about their lack of ability to succeed. One participant's ninth-grade teacher told her, "Don't try to pretend that you actually have the mental ability to do math." Fortunately, participants also told stories of teachers whose faith in their students never faltered, and who were able to convince their students of their abilities and take them on to succeed in mathematics. As a result, participants viewed good teachers as ones who believe in their students:

- . . . So I shared my secret method for quick addition [but] . . . I asked Mr. M to promise not to tell anyone because I wasn't supposed to know things because I was brown and a girl. [Mr. M] laughed and laughed and pretty soon we were both laughing hard. He said, "Some very stupid people made you believe this. It isn't true. It's a joke on them." With this teacher's encouragement I went from fourth grade to sixth, skipping fifth grade, never giving it a thought until now.
- I realized that someone believed in me, and soon it became my mission to prove Mrs. S right. I promised myself that I would not let her down. My motivation to please led me to believe that I could excel in anything.

With their words and actions, teachers through the years taught participants lessons about their ability to do mathematics. Fondly remembered teachers were those who bolstered participants' belief in their capabilities as mathematics learners through their own unwavering support.

2. Good teachers drive their instruction by their goal of student learning.

Participants shared stories of practice they perceived as ineffective, led by teachers who ignored students' questions, needs, and progress to instead base decisions on other factors:

- The [fourth-grade] teacher knew that I desperately needed extra help in math, but she never came to aid or facilitate my understanding of the subject.
- I decided to try trigonometry in my senior year, but was turned off by the material and teacher. I was so confused and did not comprehend the subject at all. The teacher went at a fast pace and did not have the time to stop to assist students.

Fortunately, participants also shared instances of teachers who served as their role models by basing their instructional decisions solidly on students' learning. Some teachers would not "go on" until every student understood. Some would spend their own time before and after school, during lunch, or on weekends in order to help students understand. Some would use multiple methods to explain concepts when the first method was ineffective. Some would advocate for their students within the school setting. Each of the actions that teachers took helped cement participants' commitment to placing students and their understanding at the center of their instructional decisions. A quote from one participant represents the perception that good teachers are driven by student learning goals:

> In high school the teacher who heightened my love for math was my geometry teacher. She explained everything precisely and she had a lot of patience. She made lectures easy and was very helpful. She offered help before and after school. If I had any questions about problems that I missed during the exam or homework, she was always willing to offer help before or after school.

3. Good teachers teach for conceptual understanding.

Data presented earlier indicate that most participants experienced a mathematics curriculum that focused on procedural competence (85 percent of the essays discussed procedural proficiency at the elementary level). Although proficiency is an important goal, many participants were frustrated by the dearth of answers they received from their teachers to the question "Why?" Some participants experienced the high point of their mathematical understanding when in their college mathematics course for teachers they finally learned the conceptual underpinnings of the elementary curriculum. (Thirteen percent of the essays discussed a clear focus on conceptual understanding in college instruction, second only in prevalence to the elementary level's 23-percent inclusion of conceptual understanding as a focus.) Participants cited strategies such as pictures, concrete materials, clear explanations, effective use of direct instruction, and multiple means of explanations as supportive of their goals of conceptual understanding.

4. Good teachers use methods that are interesting and engaging to students.

Participants appreciated strategies that moved beyond text and workbook activities to include activities such as songs and chants, games, simulations, and projects. Although participants infrequently mentioned such strategies in their own experience (for example, 2 percent of the essays mentioned experiences with centers, 2 percent included memories of writing in mathematics, and 8 percent recalled projects in mathematics), these approaches were highly valued by participants. They appreciated teachers who sparked a love for the subject matter by capitalizing on students' outside interests and students' preferences for enjoyable, engaging activities. Although some strategies were equally praised by participants, others (namely speed competitions for multiplication drill) were met with highly mixed reviews. Seemingly, good teachers listen to their students and select teaching strategies that quench students' particular preferences and needs for interesting activities.

5. Good teachers create settings in which students feel safe to take risks.

Participants shared stories of teachers in whose classrooms they felt nervous and worried about making errors. Public humiliation, often during recitations of mathematics facts or while doing problems on the board, figured prominently into participants' views of unsafe learning environments. Conversely, they told stories about teachers who created classrooms in which "there were no stupid questions," in which students felt motivated to try, and in which intellectual progress was fueled by teachers' attention to students' attitudes toward mathematics as well as their performance. Some participants set their goals as mathematics teachers based on their experiences with risk taking in the mathematics classroom:

- It is because of [my negative] experiences that I plan on doing my best to make my students look at math not with a deep, dark, sinking feeling in the pit of their stomachs, but as something more positive such as a puzzle. I plan on finding ways to allow my students the chance to enjoy math, not dread it. I just hope I can instill a positive attitude toward mathematics before someone like one of my old professors gets a hold of them.
- I will never, ever humiliate a student because they are struggling with a subject. Nor will I ever pinpoint a student to help in front of the class unless I know he or she can be successful. I have tried to turn my bad experiences into a learning one for myself and hopefully be a positive force in my students' lives.

6. Good teachers show the connections between mathematics and other facets of life.

Just 15 of 144 participants (10 percent) told of connections between mathematics and other aspects of life. Universally, participants viewed the real-life connections they cited as positive. A few examples include, at the elementary level, playing store to learn to balance checkbooks, and at the junior high level, engaging in an egg simulation in which students were required to make mathematical projections about their "babies'" needs and schedules. One participant pleasantly recalled her high school teacher: "He made math real to me by relating it to everyday life and real-world situations. This teacher made me feel like a winner, and this was a very new feeling for me."

In sum, despite the fact that most participants' mathematics histories had high points and low points, their experiences allowed them to build useful ideas that can positively affect how they teach mathematics in the elementary classroom.

Implications: Reflecting on Classroom Practices

This study analyzed outcomes of mathematics instruction by examining preservice elementary teachers' memories of earlier experiences. Memory does not provide an exact or objective accounting of actual events. Recollections can vary from actual events substantially, and they can vary across individuals. Memory also reflects a person's interpretation of events at a particular point in time. Clearly, memory is created as individuals use their own frames of references to interpret events. Perhaps precisely for this reason, student memories

Figure 3

Reflecting on classroom mathematics practices

1. How would your students describe mathematics to a person from far away?

2. Where would your students say they learn and use mathematics?

3. Which strategies do you and your school use to recognize the mathematics that occurs at home? How do you build bridges between home and school through mathematics?

4. Do students realize the importance of conceptual understanding in your classroom? What have you done in recent lessons to help students focus on conceptual development? What did you say the last time a student asked, "Why?"

5. Do students have opportunities to sense the importance of problem solving in realistic settings? When is the last time you had students solve an interesting, ill-defined problem? Does the students' memory match your memory?

6. Which recent lessons can you cite that focused on discovery learning or included hands-on and cooperative learning strategies? How many examples might your students be able to cite?

7. How well do your students see mathematics in school as connected to their real lives? How do you encourage them to build that connection?

8. How do you know that all students are interested in the same kinds of activities that you are? For example, how many of your students would say they like playing competitive games to reinforce basic facts? How many would not?

9. If your students responded anonymously to the question "Which students does your teacher think are good at mathematics?" what might they say? What strategies do you use to help all your students view themselves as competent mathematical learners?

10. How many of your students would agree with the statement "In math, my teacher checks to see whether I understand and works with me until I do" or with the statement "My teacher uses lots of ways to help students understand"?

can serve as a powerful data source for teachers and teacher educators alike as they reflect, from varied perspectives, on the effects of their instruction. By examining student memories and perspectives, teachers and teacher educators can gain insights into such factors as—

- the range of student reactions to instruction;
- the factors to which students attend during instruction;
- the unintended outcomes of instruction; and
- the conclusions that students draw, not just from one classroom experience but from the place of that experience within the context of many experiences collected over time.

Although memories are fallible, they shape our views of who we are and what we think is possible in mathematics. Because memories shape our frames of reference, they often affect actions. They shape students' actions, for example, by constraining or fueling the mathematics courses they pursue. They shape teachers' actions by providing functional definitions of what counts as mathematics and by providing de facto prescriptions for how it should be taught. Fallible or accurate, memories play a role in how students approach mathematics and in how teachers teach it.

The most powerful implication of this study is that teachers and teacher educators must become aware of their students' perspectives if they are to incorporate students' background knowledge effectively into their instruction. As indicated both in the research literature and in the current study, life experiences in mathematics can provide both positive, rich notions and negative, limiting notions about what mathematics is and how it should be learned. If students view mathematics purely as the practice of routine algorithms, teachers will need to help them deconstruct those notions. If teacher educators discover that their students enter the methods classroom (as did participants in this study) with strongly developed, although intuitive, understandings of the importance of student learning as the driving force for instruction, teacher educators have the wonderful opportunity to build on their students' existing understandings and address misunderstandings. Teachers and teacher educators must seize every possible opportunity to reflect on their practice through their students' notions and memories of mathematics.

Teachers and teacher educators can positively influence students' memories and learning by considering their instructional purposes and their purposes for gaining information on student perspectives. They can continuously search out students' past experiences and memories to reflect on their practice. Simple data-gathering techniques can even help teachers of young students gather information about the mathematics lessons that students are learning. Examples include class discussions ("How does your family use mathematics?"), drawings ("Draw a picture of you and mathematics"), and journal entries ("What are the important things to know about mathematics?"). Teacher educators may use questions such as those posed in the introduction of this article or use the prompts posed to participants in this study. Classroom teachers may, instead, present questions such as those in **figure 3** to fuel inquiry into students' perspectives.

Through continuous reflection on their instructional practices, and through ongoing dialogue with their students, teachers and teacher educators can maximize their power to positively influence students' learning—and the far-ranging, long-lasting lessons they teach.

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This paper is based on a study presented at the annual meeting of the American Educational Research Association in April 2004. ▲