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Why Students with Special Needs Have Difficulty Learning Mathematics and What Teachers Can Do to Help

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THE NATIONAL COUNCIL OF TEACHERS OF Mathematics advocates a balanced approach of teaching both procedural and conceptual knowledge (NCTM 2000). In practice, however, students with special needs often receive a great deal of algorithmic instruction because mastering algorithms is what we "see" them struggle with the most. Even with a heavy dose of algorithmic instruction, many of these students still have difficulty performing algorithms efficiently. Furthermore, without developing conceptual understanding while learning algorithms, these students will never understand foundational mathematical concepts.

To help these students, we need to understand why they have difficulty learning mathematics in the first place. Using this knowledge, we can then implement effective instructional strategies and assess their impact on students' understanding. The purpose of this article is to give readers a picture of why many students with special needs have difficulty learning mathematics and how specific instructional strategies can meet the mathematical learning needs of these students.

Barriers to Learning Mathematical Concepts and Skills

STUDENTS WHO STRUGGLE TO LEARN MATHEmatics, both those with identified disabilities (e.g., learning disabilities, attention deficit-hyperactivity disorder, and mental disabilities) and those who have not been identified as having such disabilities, struggle to learn mathematics for a variety of reasons. One particular source of difficulty for these students is that they may possess one or more learning characteristics that prevent them from learning mathematics as efficiently as their peers without learning problems (see, for example, Miller and Mercer [1997]). The following paragraphs describe four characteristics that may affect the development of mathematical understanding.

Attention problems

Students with attention problems either "miss" important information as it is presented during instruction, or they do not attend in a meaningful way to essential cues when problem solving. A common misconception about attention deficits is that students do not focus on anything. In actuality, just the opposite is true. These students often focus on everything that assails their senses. Because of this heightened attention to stimuli, students tend to be distracted from essential learning or problem-solving cues. Attention problems affect procedural knowledge when students miss an important step in a procedure. For example, students may miss the "subtract" step in the long-division process, which is described as "divide, multiply, subtract, bring down." Attention difficulties can also affect problem solving that requires using parts of a situation to develop hypotheses. For example, using a graphing calculator, students can examine the effects of changing the coefficients in the quadratic equation $y = ax^2 + bx + c$. Students with attention problems attend to every slight modification in the graph and, thus, have difficulty isolating the ways in which the graph changes in general. In either example, attention problems result in gaps in a student's knowledge base.

Cognitive-processing problems

Although students may have adequate vision and hearing, they may still have difficulty "interpreting" what they see or hear. For example, a student may accurately see a mathematics problem on the chalkboard or overhead projector. The student may also be able to write numbers, symbols, and words correctly from memory. However, when the student tries to write what he sees on a chalkboard or transparency, he writes it inaccurately. Such errors occur because of a disruption in the student's central nervous system that alters the message received from the visual system (eyesight) as it moves to the motor system (writing). The same situation can occur with auditory information. The student may hear the teacher say, "$3a + 2a = 25$," but the student might actually process "$2a - 3a = 52$," and the second problem is what the student thinks the teacher said. When the student responds incorrectly to the problem posed or appears confused, the teacher may assume that the student does not understand or is not paying attention. In reality, the student may understand how to solve the problem $3a + 2a = 25$, but that problem is not the one that the student processed cognitively.

Faulty memory retrieval is especially problematic

When the teacher relies primarily on one means of sensory input when modeling a mathematical concept or skill, for example, visual input, through use of a chalkboard or overhead projector, students who have cognitive-processing problems that occur through that particular sensory modality will not process the elements of the concept in the manner the teacher intends. As a result, what these students have learned, or processed, does not fit the teacher's expectations.

Memory problems

When problem solving, successful learners are able to retrieve essential information from memory accurately and quickly. In contrast, students who have memory problems often have difficulty retrieving the necessary information accurately or efficiently even though they may have successfully stored the information at one time. For example, a student may have learned that the decimal 0.25 and the fraction 1/4 are equivalent. When converting 1/4 to decimal form, however, the student may mistakenly retrieve from memory the decimal 0.52 or 0.75. At other times, the student may need a prolonged period of time to make the correct association. Faulty memory retrieval is especially problematic when students are confronted with multistep tasks, such as doing long division, solving algebraic equations, or engaging in problem solving, because these students often "mix up" or omit certain steps.

Metacognitive deficits

Metacognition involves the ability to apply appropriate learning strategies, to evaluate their effectiveness, and to change strategies when current ones are not successful. Some students are able to discover strategies independently or detect implicit strategies that they see others use. Students who have metacognitive deficits may not even be aware that others are using strategies to successfully complete the task at hand. For example, suppose students are asked to draw a rectangular dog pen with the largest possible area given a certain amount of fencing, that is, the perimeter would remain constant. Students with metacognitive deficits may be unable to list possible rectangles with the given perimeter to determine the one with the largest area. They may be unaware that they could list the possibilities systematically. Metacognitive deficits become more evident as students are expected to apply strategies they have learned to new situations, concepts, or skills. For example, if students with metacognitive deficits have been explicitly taught how to make systematic lists for such tasks as drawing the rectangular pen, they may not automatically apply this strategy in a new context, such as listing a sample space in probability tasks, because they do not realize that a systematic list is an appropriate strategy for such tasks.

Effective Teaching Practices for Students with Special Needs

A GROWING RESEARCH BASE PROVIDES EDUCATORS with a solid foundation for helping special needs students learn mathematics (for example, Miller and

Mercer [1997]). Some of these instructional strategies include the following:

- teaching in authentic and meaningful contexts;
- directly modeling both general problem-solving strategies and specific learning strategies using multisensory techniques;
- ensuring that the sequence of instruction moves from the concrete, to the representational, to the abstract;
- giving students opportunities to use their language to describe their mathematical understandings;
- providing multiple practice opportunities to help students use their developing mathematical knowledge and build proficiency; and
- continually monitoring students' performance and offering meaningful feedback in the form of performance charts.

These instructional strategies are effective because each strategy incorporates teaching practices that accommodate the previously discussed learning characteristics of special needs students.

The following vignette illustrates how several of these effective instructional strategies can be used to help students who have special needs develop mathematical understanding and skills. The vignette is based on one author's experiences working with four eighth-grade mathematics teachers and their students as part of a study designed to evaluate the effectiveness of the *Building Algebra Skills* series. This series is a prealgebra and beginning algebra instructional program for students who have mathematics learning problems (Allsopp 1999). A significant percentage of students in these eighth-grade classes, including students with identified learning disabilities, were experiencing difficulty learning the mathematical concepts being taught. Many of these students were failing or had scored in the thirtieth percentile or lower on the latest state assessment in mathematics. In particular, these students were having difficulty interpreting story problems algebraically. Using the six instructional strategies listed above resulted in an increase in the students' ability to independently solve one-variable algebra problems from a rate of 40 percent to a rate of over 85 percent (Allsopp 1997).

This vignette highlights one of the classrooms in which the teacher, Ms. Dimarco, used a concrete-to-representational-to-abstract sequence of instruction with explicit teacher modeling of problem-solving strategies; taught in authentic and meaningful contexts; gave students multiple opportunities to practice solving algebra story problems and to use language to describe their mathematical understanding; and helped students to recognize their progress by encouraging them to chart their performance. "Michael," a student with cognitive-processing difficulties, metacognitive deficits, and memory-retrieval problems, as well as a long history of difficulties learning mathematics, is the main focus in this vignette.

Identifying the problem and applying instructional strategies

Michael's experiences with solving story problems had been unsuccessful. In fact, he frequently did not even attempt to solve the problems. At times, he impulsively scribbled an equation and solution that had little relation to the problem to be solved. Ms. Dimarco was amazed that Michael often believed that he had successfully solved a story problem even when his solution was impossible. Clearly, Michael did not understand the algebraic concepts in the problems.

To help Michael develop a deeper understanding of algebraic concepts and become more adept at solving algebra story problems, Ms. Dimarco decided to use several instructional strategies with him and other students who were struggling. First, she decided to make the story problems more meaningful by incorporating authentic contexts. She wrote story problems that touched on the interests of students of Michael's age (e.g., buying CDs of their favorite music groups from the local music store). She also used her students' names in the story problems. To develop the problems, Ms. Dimarco invited Michael and other students to describe what kind of CDs they liked and what they do when they go to buy CDs. Together, the students and Ms. Dimarco wrote several story problems that incorporated these student experiences (see **fig. 1**). Ms. Dimarco saw a distinct difference in her students' interest in these story problems compared with those found in their mathematics textbook.

Because of her concerns about the students' metacognitive deficits, Ms. Dimarco decided to

> Michael and Maria went to the music store to buy some of their favorite CDs. Michael bought 3 rap CDs, including Dizzy Z, Rapman, and Little Ivey, and Maria bought 4 rhythm-and-blues CDs, including Jamie Smith, Luke Issac, The Blues, and Terrance King. Each CD cost the same, and together, Michael and Maria spent $42. How much did each CD cost?

Fig. 1 Sample story problem generated by Ms. Dimarco and her students

1. **F**ind what you are solving for.
2. **A**sk yourself what is the important information.
3. **S**et up the equation.
4. **T**ake the equation and solve it.
5. **D**iscover the variable, the operations, and what the left side of the equation equals.
6. **R**ead the equation and combine like terms on each side of the equation.
7. **A**nswer the equation and check.
8. **W**rite the answer for the variable and check the equation.

Fig. 2 FASTDRAW strategy for solving algebra story problems

teach them a strategy that they could use to solve algebra story problems. She thought about how she might solve such problems herself, then divided the process into steps. She knew that teaching strategies in the form of mnemonic devices is effective for students with special needs because it helps them to retrieve problem-solving steps from memory independently and efficiently. Ms. Dimarco's solution was an eight-step process for solving algebra story problems, written as a mnemonic device, FASTDRAW, to help her students remember the steps (see **fig. 2**).

Keeping in mind that some of her students had cognitive-processing problems, Ms. Dimarco used more than one means of sensory input during instruction. She made a visual display of the strategy on poster board and gave students paper copies of the strategy. She described each step and asked students to think about the meaning and recite it in their own words. Encouraging students to use their own words to describe each step was especially helpful because it gave students a meaningful way to process their understanding and allowed Ms. Dimarco to evaluate each student's thinking.

Ms. Dimarco then explicitly modeled the strategy for solving the problems that she and the students had written. For example, when modeling the first step in FASTDRAW, Ms. Dimarco read the step while pointing to it, then spoke her thoughts aloud as she read the story problem and looked for a phrase that posed the problem to be solved. When she discovered the question "How much did each CD cost?" she asked the students whether this question represented a problem to be solved. When the students agreed that it did, Ms. Dimarco underlined the question and asked her students to do the same on their copies of the problem. Ms. Dimarco reinforced what students had just experienced by asking them to describe what she did and, more important, why she did it. Michael volunteered that underlining the question, "helps me better see what

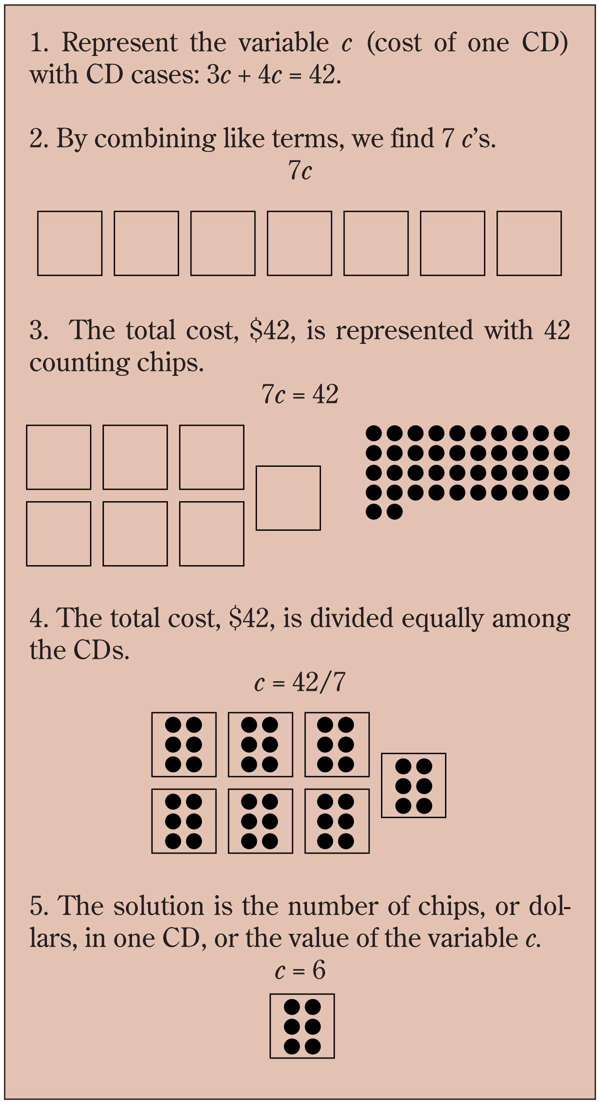

Fig. 3 Using concrete materials and DRAW steps to solve an equation

the problem is that I need to solve, and I have a way to go back to it if I forget."

Ms. Dimarco continued to model each step, making sure to use visual cues, such as displaying the strategy and story problem; auditory cues, such as thinking aloud; and kinesthetic cues, such as pointing to the steps. After she and her students developed an appropriate equation to solve the problem, Ms. Dimarco modeled the DRAW steps. She incorporated the use of simple concrete materials to help students visualize the algebraic process for finding how much each CD cost, which was the variable, or unknown, c. She brought in some of her own CD cases to represent CDs and used counting chips to represent dollars (see **fig. 3**). As Ms. Dimarco modeled the steps with concrete materials, she encouraged students to do the same with the materials at their desks. She continued to ask them to describe their actions as they completed each step of DRAW.

Realizing improvements in students' abilities and confidence

Students continued to practice using the FAST-DRAW strategy and concrete materials with other story problems. At times, they practiced independently, and at other times, they worked in peer-tutoring groups. Ms. Dimarco monitored students as they practiced and offered specific corrective feedback. She also made specific positive comments to students about their efforts and accuracy. Ms. Dimarco decreased her direction as Michael and the others demonstrated that they understood the process. The students had many opportunities to practice and apply the feedback that they received on their developing understanding of algebra. As a result, they became proficient at solving simple story problems independently. Later, Ms. Dimarco taught them to draw simple pictures to represent the concrete materials in the problems, such as squares for CD cases and dots for dollars. Eventually, students chose to try solving problems without the use of concrete materials or drawings.

Throughout the instructional process, Ms. Dimarco encouraged students to chart their progress by plotting the number of problems they solved correctly and incorrectly. These "learning pictures" offered Michael and other students a visual representation of their progress. For the first time, they could actually "see" their progress, which they could not do with number or letter grades. An example of Michael's "successful" learning picture is shown in **figure 4.**

The meaningful context, the use of concrete materials and drawings in conjunction with an explicitly taught strategy for solving problems, and the process of putting his understanding in his own words helped Michael to connect basic concepts he had learned previously to a process for solving problems algebraically. With continued practice, specific feedback from Ms. Dimarco, and a visual representation of his progress, Michael's mathematical understanding and confidence grew. At the end of several weeks, Michael remarked that for the first time in his memory, he really understood mathematics. He said that before this experience, mathematics had never been clear to him. Michael's success did not occur by chance. Ms. Dimarco chose to use instructional strategies that effectively accommodated the learning characteristics that previously had been barriers to learning for Michael and his peers. This instruction increased her special needs students' understanding and gave them a process to use for independent work.

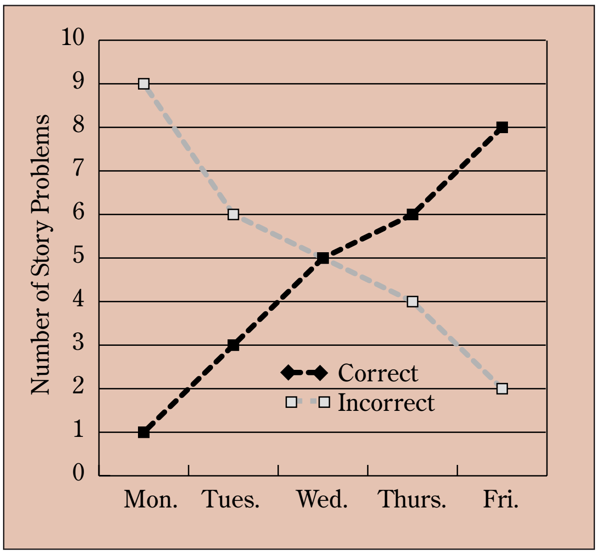

Fig. 4 Michael's "successful" learning picture (number of correct problems increases as number of incorrect problems decreases)

Conclusions

ONE OF THE MAIN PREMISES OF THE MOST current reform efforts in mathematics education is that educators want to empower students mathematically to ensure that they are confident and successful in exploring and engaging in significant mathematical problems. Furthermore, the Equity Principle in *Principles and Standards for School Mathematics* states, "Mathematics can and must be learned by *all* students" (NCTM 2000, p. 13). Unfortunately, this principle is more easily established in the abstract than put into practice. Many teachers leave their teacher education programs with minimal instruction on effective strategies for working with children who have special needs. Additionally, effective instructional practices for students with special needs are poorly integrated into many mathematics textbooks and series and, therefore, are not implemented systematically in classrooms. The result for these students is instruction that does not meet their learning needs and offers the possibility of only limited success.

"Learning pictures" offer students a visual representation

According to the spirit of the NCTM's *Standards,* we need to begin instruction on our students' current levels of understanding before they will understand the mathematics we want them to learn. Otherwise, mathematics becomes something that students attempt to merely memorize, and relying on memorization places many students who have special needs in a precarious situation. A growing research base provides educators with a solid foundation for effectively teaching mathematics to these students. Understanding first why special needs students have difficulty learning mathematics puts classroom teachers well on the road to helping these students see the joy in doing mathematics.

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Visit the *Mathematics Video Instructional Development Source (MathVIDS)* Web site at etv.jmu.edu/mathvids to learn more about how these instructional strategies can be used to effectively teach mathematics to students who have learning problems. *MathVIDS* includes video models of teachers implementing these strategies, as well as instructional plans that demonstrate how to use these strategies to teach particular mathematics concepts. *MathVIDS* is password-protected, but users can take the *MathVIDS* tours without entering the required password. After taking the *MathVIDS* tours, interested persons can contact David Allsopp for password information at dallsopp@tempest.coedu.usf.edu or 4202 East Fowler Ave., EDU 162, University of South Florida, Tampa, FL 33620. □