A decorative graphic featuring various colored circles and dashed lines. In the top left, there is a large teal ring and a smaller teal circle. In the top right, there is a large lime green circle and a smaller green circle. In the bottom left, there is a large green circle with a white dot and a smaller yellow circle. In the bottom right, there is a large yellow ring and a smaller orange circle. A dashed blue line curves around the central text area.

Integrating the Standards for Mathematical Practice Into Your Daily Lessons

Part 1 - Standards 1, 2, 3, 6

Dr. Janet Stramel

Hello!



Fort Hays State University

- Mathematics Methods
- Concepts of Elementary Mathematics

National Board Certified Teacher

- Early Adolescence Mathematics

Middle School Mathematics Teacher

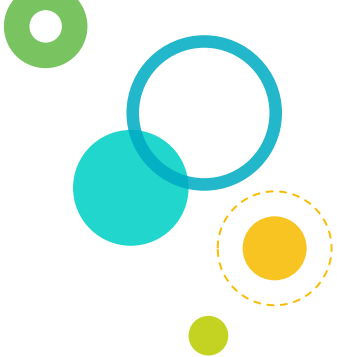
I am Dr. Janet Stramel.

I am here because I love teaching mathematics and love to give presentations.

You can find me at jkstramel@fhsu.edu

A decorative graphic on the left side of the slide consists of several overlapping circles and rings in various colors: pink, orange, teal, light blue, and lime green. Some circles have dashed outlines, and some are solid. The shapes are arranged in a vertical, somewhat scattered pattern.

Session Outline

A decorative graphic on the right side of the slide consists of several overlapping circles and rings in various colors: teal, yellow, and lime green. Some circles have dashed outlines, and some are solid. The shapes are arranged in a vertical, somewhat scattered pattern.

The Standards for Mathematical Practice reflect what it means to “do mathematics.” This session will give you suggestions for incorporating them into your everyday mathematics instruction.



Standards for Mathematical Practice

Mathematically proficient students—

- **explain** to themselves the meaning of a problem and look for entry points to its solution.
- **make** sense of quantities and their relationships in problem situations.
- **use** assumptions, definitions, and previously established results in constructing arguments.
- **apply** the mathematics they know to solve problems arising in everyday life, society, and the workplace.
- **consider** all available tools when solving a mathematical problem.
- **communicate** precisely to others.
- **look** closely to discern a pattern or structure.
- **notice** if calculations are repeated and look for general methods and shortcuts.

Standards for Mathematical Practice

*Mathematically proficient students **do not** strictly...*

- **listen** while the teacher provides direct instruction for the entire class period.
- **copy** notes word for word from the board.
- **memorize** procedures for solving problems and duplicate exact replicas for homework and assessments.
- **sit** quietly throughout the entire lesson with minimal peer interaction.

CCSS Mathematical Practices

OVERARCHING HABITS OF MIND

1. Make sense of problems and persevere in solving them
6. Attend to precision

REASONING AND EXPLAINING

2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others

MODELING AND USING TOOLS

4. Model with mathematics
5. Use appropriate tools strategically

SEEING STRUCTURE AND GENERALIZING

7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

Standards for Mathematical Practice



Standards 1 and 6
“Overarching Habits of Mind”

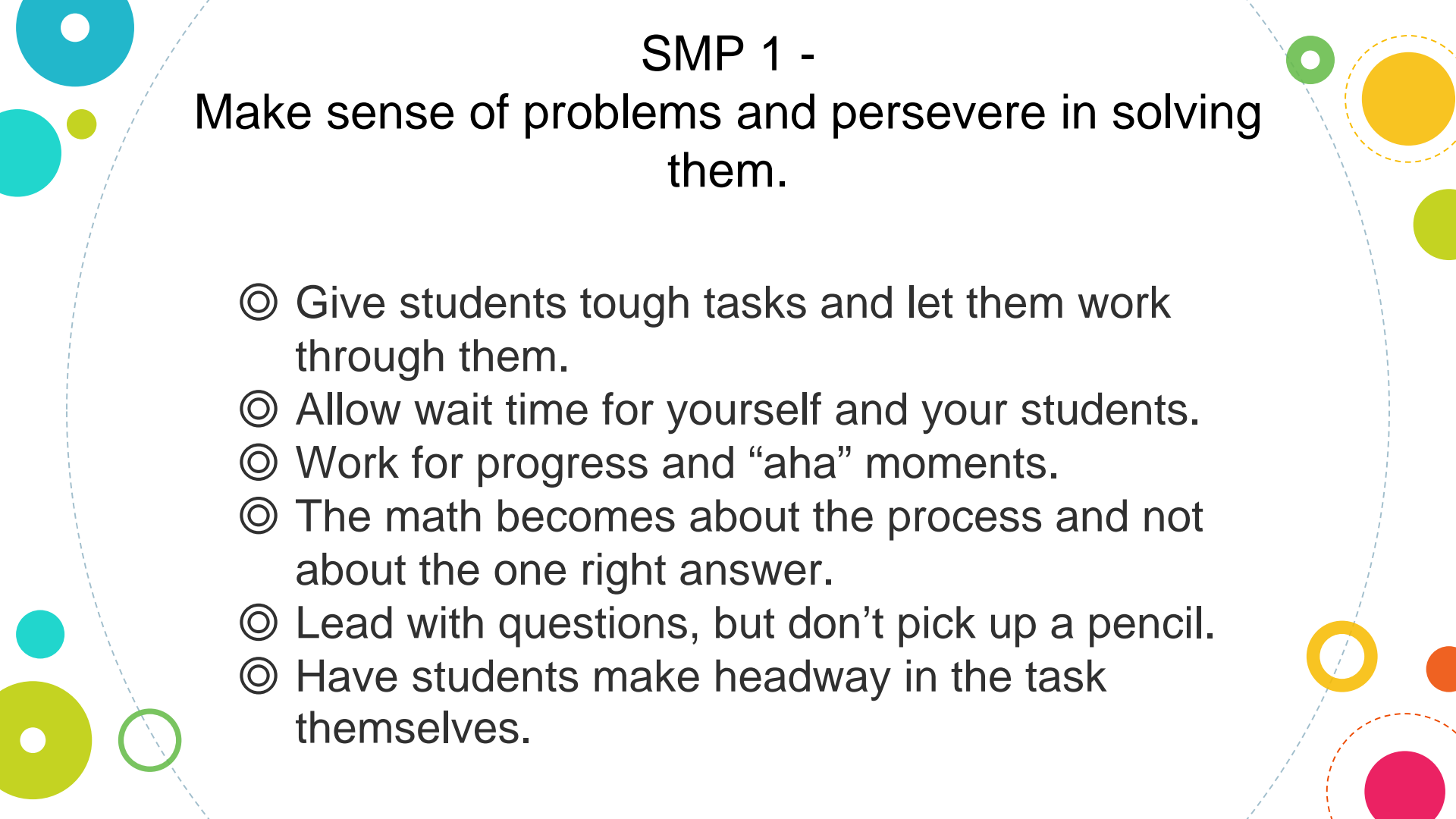


SMP

1

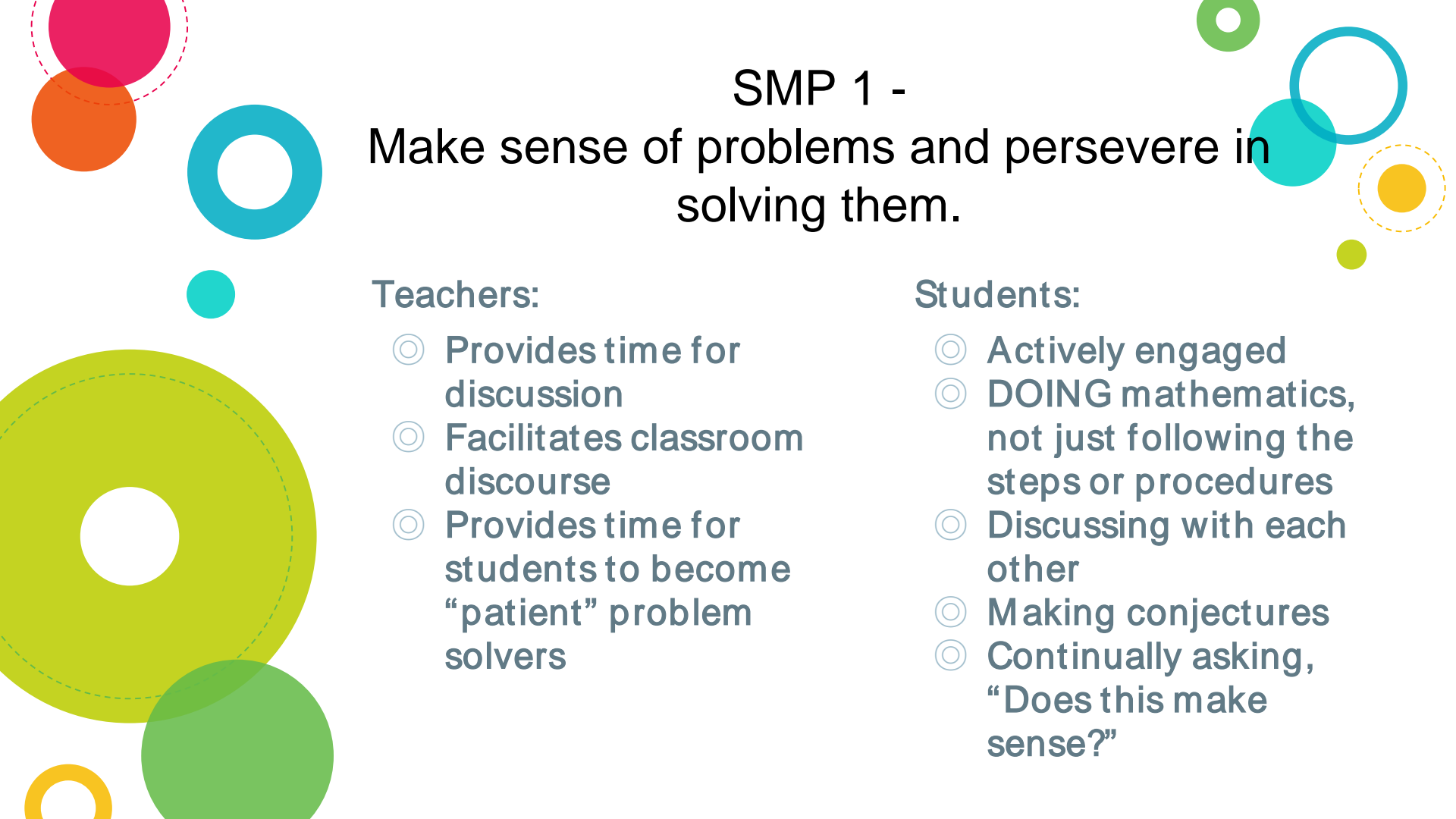
Make sense of problems and
persevere in solving them

- Mathematically proficient students explain to themselves the meaning of a problem and look for entry points to its solution.



SMP 1 - Make sense of problems and persevere in solving them.

- ◎ Give students tough tasks and let them work through them.
- ◎ Allow wait time for yourself and your students.
- ◎ Work for progress and “aha” moments.
- ◎ The math becomes about the process and not about the one right answer.
- ◎ Lead with questions, but don't pick up a pencil.
- ◎ Have students make headway in the task themselves.



SMP 1 - Make sense of problems and persevere in solving them.

Teachers:

- ⦿ Provides time for discussion
- ⦿ Facilitates classroom discourse
- ⦿ Provides time for students to become “patient” problem solvers

Students:

- ⦿ Actively engaged
- ⦿ **DOING** mathematics, not just following the steps or procedures
- ⦿ Discussing with each other
- ⦿ Making conjectures
- ⦿ Continually asking, “Does this make sense?”

A decorative graphic consisting of several overlapping circles and rings in various colors including teal, orange, yellow, pink, green, and lime green. Some circles are solid, while others are dashed or hollow. The graphic is positioned on the left and right sides of the slide, framing the central text.

SMP 1 Sample Task

SMP 1 - Questions to Develop Mathematical Thinking

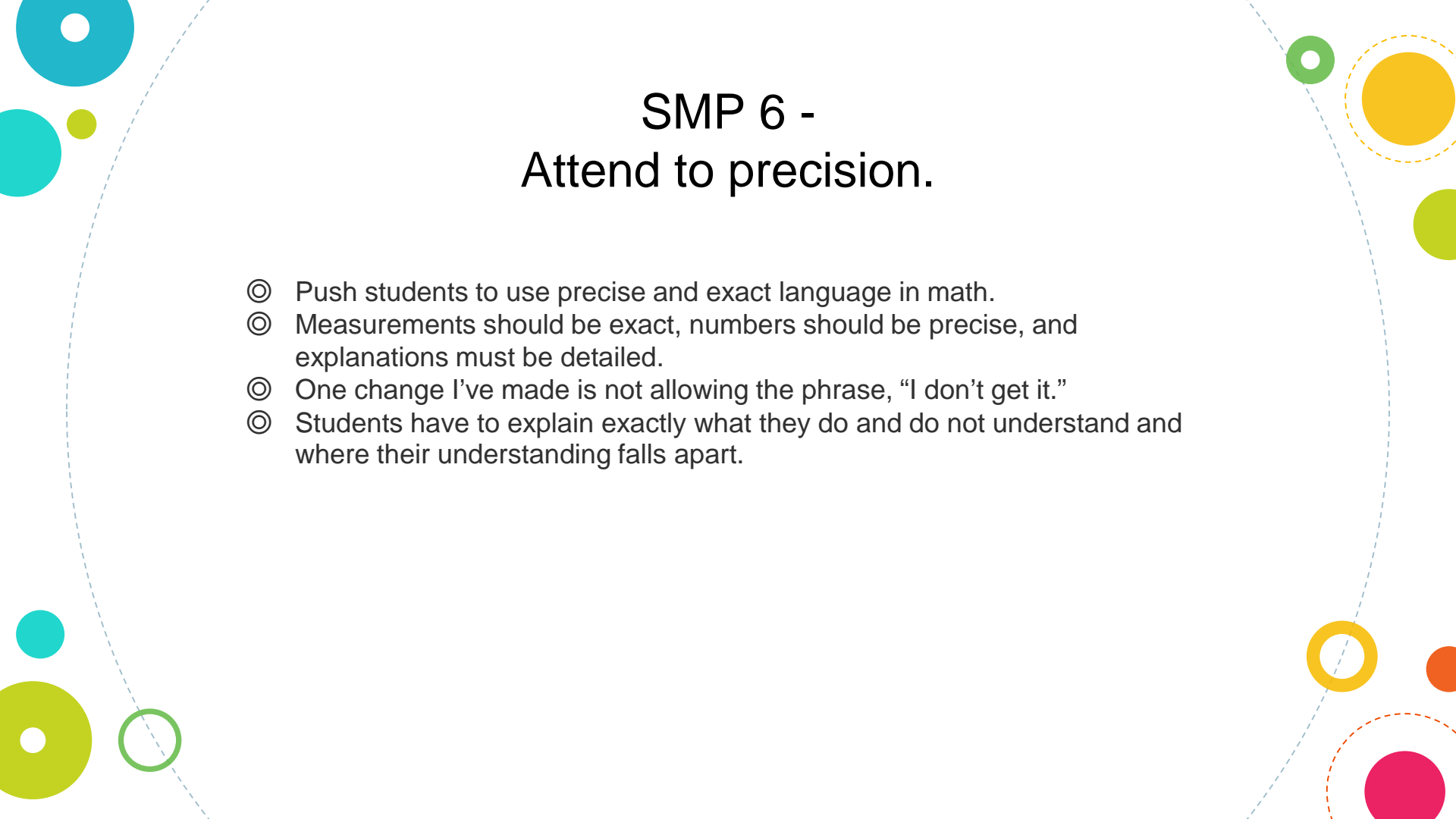
- ⊙ How would you describe the problem in your own words?
- ⊙ How would you describe what you are trying to find?
- ⊙ What do you notice about...?
- ⊙ What information is given in the problem?
- ⊙ Describe what you have already tried. What might you change?
- ⊙ Talk me through the steps you've used to this point.



SMP 6

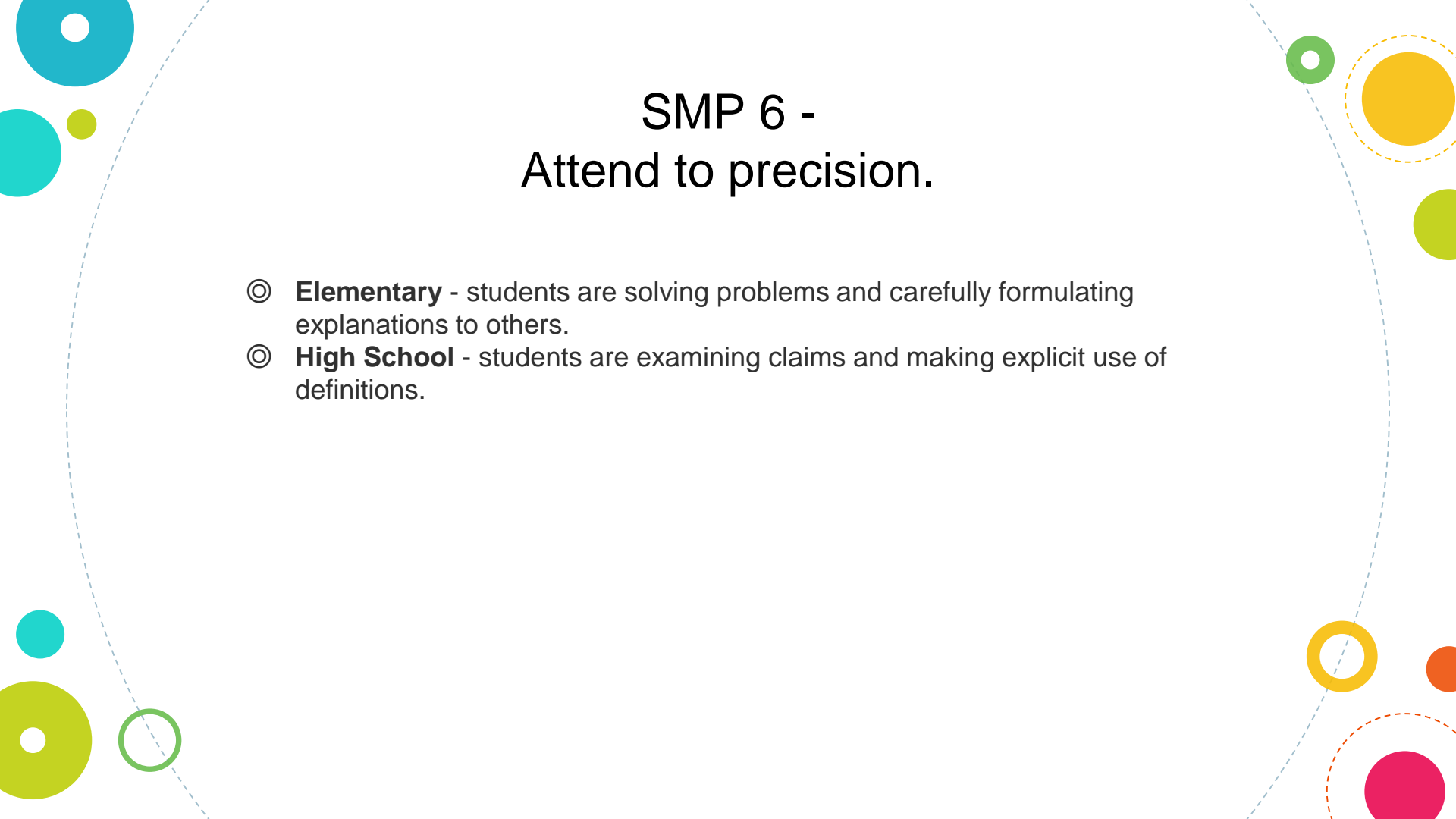
Attend to precision

→ Mathematically proficient students communicate precisely to others.

A decorative graphic consisting of a large, light blue dashed circle that frames the central text. Various colored circles (teal, yellow, green, orange, pink) are scattered around the perimeter, some solid and some with dashed outlines.

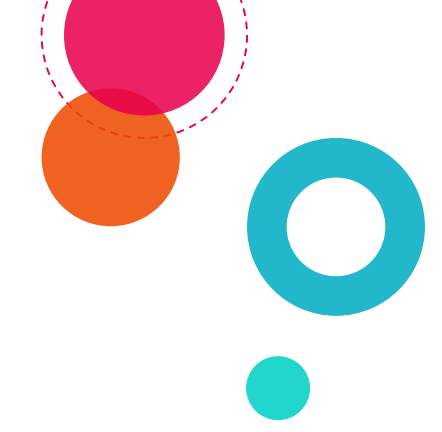
SMP 6 - Attend to precision.

- ⦿ Push students to use precise and exact language in math.
- ⦿ Measurements should be exact, numbers should be precise, and explanations must be detailed.
- ⦿ One change I've made is not allowing the phrase, "I don't get it."
- ⦿ Students have to explain exactly what they do and do not understand and where their understanding falls apart.

A decorative graphic consisting of a large, light blue dashed circle that frames the central text. Various colored circles (solid and hollow) in shades of teal, yellow, green, and orange are scattered around the perimeter of the dashed circle.

SMP 6 - Attend to precision.

- © **Elementary** - students are solving problems and carefully formulating explanations to others.
- © **High School** - students are examining claims and making explicit use of definitions.



SMP 6 - Attend to precision

Teachers:

Students:



SMP 6 Sample Task

<https://www.illustrativemathematics.org/practice-standards/6>



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About Us

Grade	1
Domain	Operations And Algebraic Thinking
Cluster	Work With Addition And
Standard	Understand The Meaning Of The
Task	Valid Equalities?

Valid Equalities?



No Tags

Alignments to Content Standards: [1.OA.D.7](#)

[Student View](#)

Task

Decide if the equations are true or false. Explain your answer.

a. $2 + 5 = 6$

b. $3 + 4 = 2 + 5$

c. $8 = 4 + 4$

d. $3 + 4 + 2 = 4 + 5$

SMP 6 Sample Task

<https://www.illustrativemathematics.org/content-standards/7/NS/A/1/tasks/314>



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About Us

Grade	7
Domain	The Number System
Cluster	Apply And Extend Previous
Standard	Apply And Extend Previous
Task	Comparing Freezing Points

Comparing Freezing Points

No Tags

Alignments to Content Standards: **7.NS.A.1**

Student View

Task

Ocean water freezes at about $-2\frac{1}{2}^{\circ}\text{C}$. Fresh water freezes at 0°C . Antifreeze, a liquid used in the radiators of cars, freezes at -64°C .

Imagine that the temperature has dropped to the freezing point for ocean water. How many degrees more must the temperature drop for the antifreeze to turn solid?

SMP 6 - Questions to Develop Mathematical Thinking

- ⊙ How did you know your solution was reasonable?
- ⊙ Explain how you might show that your solution answers the problem.
- ⊙ Is there a more efficient strategy?
- ⊙ How are you showing the meaning of the quantities?
- ⊙ How could you test your solution to see if it answers the problem?

Standards for Mathematical Practice




Standards 2 and 3
"Reasoning and Explaining"



SMP 2

Reason abstractly and quantitatively

- Mathematically proficient students make sense of quantities and their relationships in problem situations.



SMP 2 - Reason abstractly and quantitatively

Teachers:

- ⦿ Provide a range of problems
- ⦿ Provide problems that require flexibility
- ⦿ Emphasize quantitative reasoning



Students:

- ⦿ Use varied representations
- ⦿ Decontextualize and contextualize
- ⦿ Use quantitative reasoning



SMP 2 Sample Task

<https://www.illustrativemathematics.org/practice-standards/2>

In the barnyard is an assortment of chickens and pigs. Counting heads, I get 13; counting legs I get 46. How many pigs and chickens are there?

SMP 2 Sample Task

<https://achievethecore.org/aligned/digging-deeper-into-smp-2-reason-abstractly-and-quantitatively/>



Jane wants to save \$53 to buy her cousin a birthday present. She also needs \$34 for a new pair of shoes. How much money will Jane need to save for both?



SMP 2 Sample Task

<https://achievethecore.org/aligned/digging-deeper-into-smp-2-reason-abstractly-and-quantitatively/>

There were 24 boys and 20 girls in a chess club last year. This year the number of boys increased by 25% but the number of girls decreased by 10%. Was there an increase or decrease in overall membership? Find the overall percent change in membership of the club.

**Question taken from Illustrative Mathematics



SMP 2 Sample Task

<https://achievethecore.org/aligned/digging-deeper-into-smp-2-reason-abstractly-and-quantitatively/>

A fisherman illegally introduces some fish into a lake, and they quickly propagate. The growth of the population of this new species (within a period of a few years) is modeled by $P(x) = 5b^x$, where x is the time in weeks following the introduction and b is a positive unknown base.

- Exactly how many fish did the fisherman release into the lake?
- Find b if you know the lake contains 33 fish after eight weeks. Show step-by-step work.
- Instead, now suppose that $P(x) = 5b^x$ and $b = 2$. What is the weekly percent growth rate in this case? What does this mean in every-day language?

**Question taken from Illustrative Mathematics

SMP 2 - Questions to Develop Mathematical Thinking


- © What do the numbers used in the problem represent?
- © What is the relationship of the quantities?
- © What does _____ mean to you? (e.g. symbol, quantity, diagram)
- © What properties might we use to find a solution?
- © How did you decide in this task that you needed to use...?



SMP 3

Construct viable arguments and
critique the reasoning of others

→ Mathematically proficient students use assumptions, definitions, and previously established results to construct arguments.



SMP 3 - Construct viable arguments and critique the reasoning of others

Teachers:

- ⦿ Provides opportunities for ALL students
- ⦿ Provides ample time
- ⦿ Facilitates and guides students
- ⦿ Asks useful questions

Students:

- ⦿ Makes conjectures
- ⦿ Justifies and defends all conclusions and communicates them to others



SMP 3 Sample Task

How Do You Know That $23 + 2 = 2 + 23$?

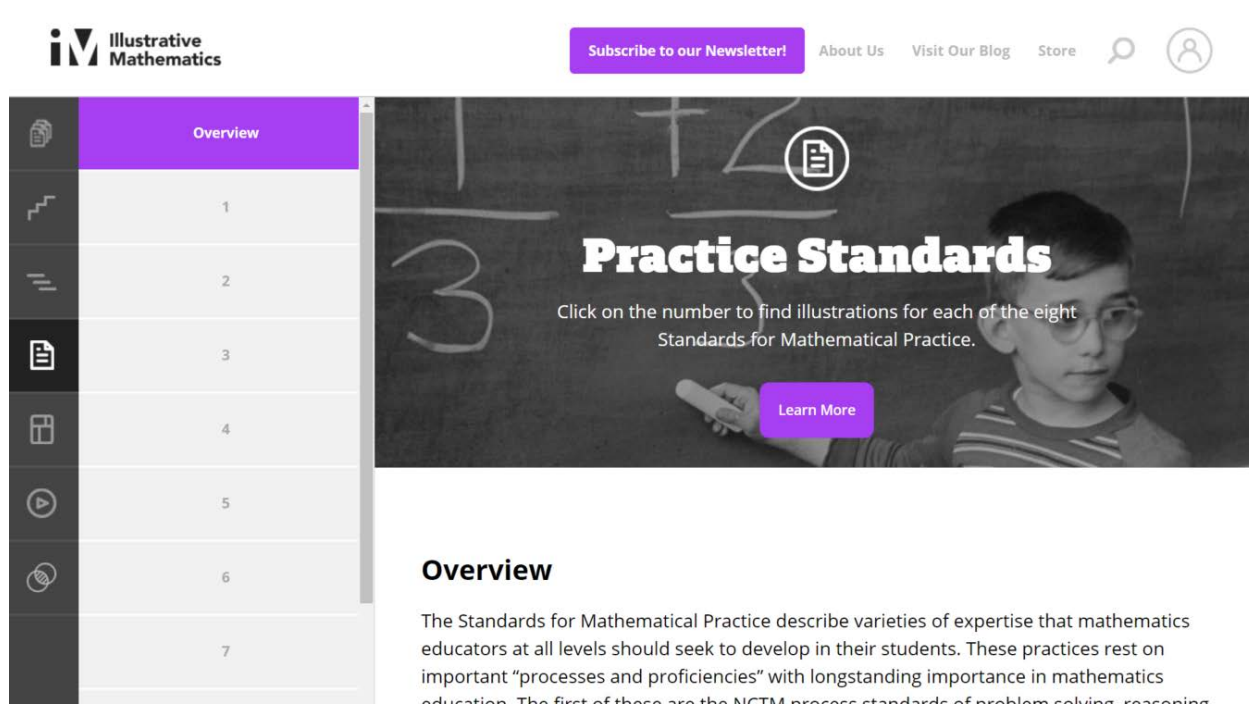
- What happens to the sum if the order of the addends is changed?
- Can we change the order of the addends and still get 25?

SMP 3 - Questions to Develop Mathematical Thinking



- © How can we be sure that...? / How could you prove that...? Will it still work if...?
- © What were you considering when...?
- © How did you decide to try that strategy?
- © How did you decide what the problem was asking you to find? (What was unknown?)
- © Did you try a method that did not work? Why didn't it work? Would it ever work? Why or why not?

Illustrative Mathematics: Practice Standards

<https://www.illustrativemathematics.org/practice-standards>



The screenshot shows the website's navigation bar with the Illustrative Mathematics logo, a "Subscribe to our Newsletter!" button, and links for "About Us", "Visit Our Blog", and "Store". A search icon and a user profile icon are also present. Below the navigation bar is a sidebar menu with icons for various content types and a list of numbered items (1-7). The main content area features a large banner with a chalkboard background, a boy holding a marker, and the text "Practice Standards" followed by a description and a "Learn More" button.

Illustrative Mathematics [Subscribe to our Newsletter!](#) [About Us](#) [Visit Our Blog](#) [Store](#)  

Overview

1
2
3
4
5
6
7

Practice Standards

Click on the number to find illustrations for each of the eight Standards for Mathematical Practice.

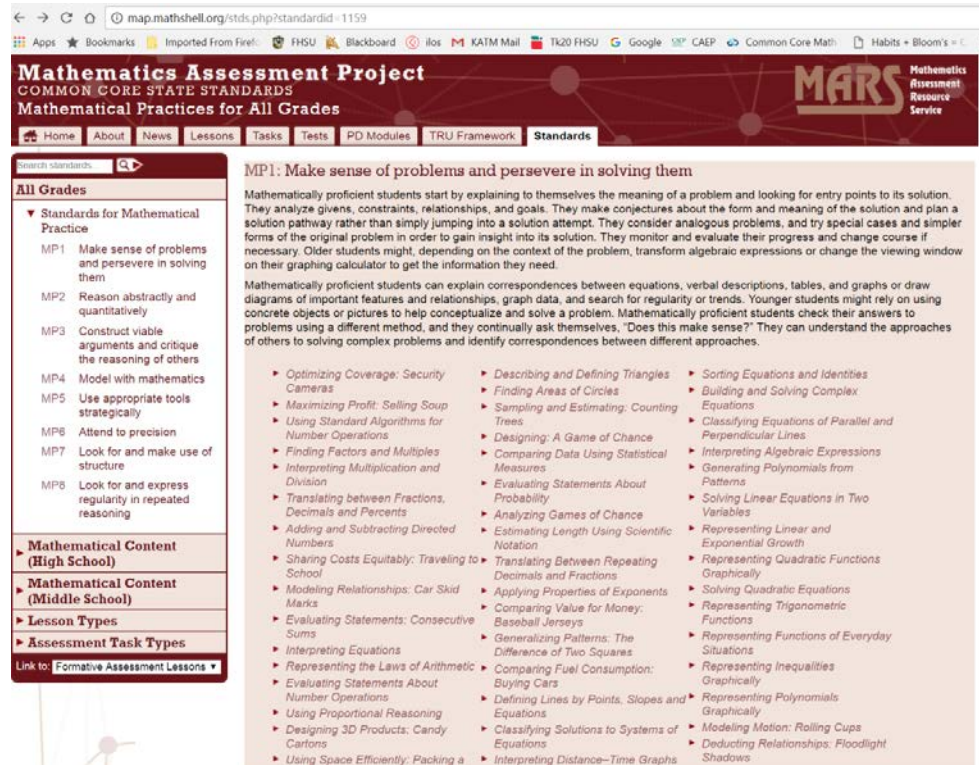
[Learn More](#)

Overview

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important “processes and proficiencies” with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning

Mathematics Assessment Project

<http://map.mathshell.org/stds.php?standardid=1159>



The screenshot shows a web browser window displaying the Mathematics Assessment Project website. The page title is "Mathematics Assessment Project" and the subtitle is "COMMON CORE STATE STANDARDS Mathematical Practices for All Grades". The navigation menu includes Home, About, News, Lessons, Tasks, Tests, PD Modules, TRU Framework, and Standards. The search bar shows "Search standards:" and "All Grades". The left sidebar lists standards for All Grades, with MP1 selected. The main content area displays the description of MP1: "Make sense of problems and persevere in solving them".

Mathematics Assessment Project
COMMON CORE STATE STANDARDS
Mathematical Practices for All Grades

Home About News Lessons Tasks Tests PD Modules TRU Framework Standards

Search standards:

All Grades

- Standards for Mathematical Practice
- MP1 Make sense of problems and persevere in solving them
- MP2 Reason abstractly and quantitatively
- MP3 Construct viable arguments and critique the reasoning of others
- MP4 Model with mathematics
- MP5 Use appropriate tools strategically
- MP6 Attend to precision
- MP7 Look for and make use of structure
- MP8 Look for and express regularity in repeated reasoning

Mathematical Content (High School)

Mathematical Content (Middle School)

Lesson Types

Assessment Task Types

Link to: **Formative Assessment Lessons**

MP1: Make sense of problems and persevere in solving them

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need.

Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

- Optimizing Coverage: Security Cameras
- Maximizing Profit: Selling Soup
- Using Standard Algorithms for Number Operations
- Finding Factors and Multiples
- Interpreting Multiplication and Division
- Translating between Fractions, Decimals and Percents
- Adding and Subtracting Directed Numbers
- Sharing Costs Equitably: Traveling to School
- Modeling Relationships: Car Skid Marks
- Evaluating Statements: Consecutive Sums
- Interpreting Equations
- Representing the Laws of Arithmetic
- Evaluating Statements About Number Operations
- Using Proportional Reasoning
- Designing 3D Products: Candy Cartons
- Using Space Efficiently: Packing a
- Describing and Defining Triangles
- Finding Areas of Circles
- Sampling and Estimating: Counting Trees
- Designing: A Game of Chance
- Comparing Data Using Statistical Measures
- Evaluating Statements About Probability
- Analyzing Games of Chance
- Estimating Length Using Scientific Notation
- Translating Between Repeating Decimals and Fractions
- Applying Properties of Exponents
- Comparing Value for Money: Baseball Jerseys
- Generalizing Patterns: The Difference of Two Squares
- Comparing Fuel Consumption: Buying Cars
- Defining Lines by Points, Slopes and Equations
- Classifying Solutions to Systems of Equations
- Interpreting Distance-Time Graphs
- Sorting Equations and Identities
- Building and Solving Complex Equations
- Classifying Equations of Parallel and Perpendicular Lines
- Interpreting Algebraic Expressions
- Generating Polynomials from Patterns
- Solving Linear Equations in Two Variables
- Representing Linear and Exponential Growth
- Representing Quadratic Functions Graphically
- Solving Quadratic Equations
- Representing Trigonometric Functions
- Representing Functions of Everyday Situations
- Representing Inequalities Graphically
- Representing Polynomials Graphically
- Modeling Motion: Rolling Cups
- Deducting Relationships: Floodlight Shadows



Other Resources of Note:

© Kansas FlipBooks

- <http://community.ksde.org/Default.aspx?tabid=5646>

© Blog by Bill McCallum

- <http://commoncoretools.me/>

© You Cubed

- <https://www.youcubed.org/>

© Debbie Waggoner

- <http://www.debbiewaggoner.com/math-practice-standards.html>

© Inside Mathematics

- <http://www.insidemathematics.org/>

Thank You!



Any questions?

You can find me at jkstramel@fhsu.edu



References

- ◎ <http://www.nctm.org/Publications/Mathematics-Teaching-in-Middle-School/Blog/What-Makes-a-Mathematically-Proficient-Student/>
- ◎ <https://www.scholastic.com/teachers/blog-posts/meghan-everette/guide-8-mathematical-practice-standards/>
- ◎ <http://thinkmath.edc.org/>
- ◎ <https://www.nwea.org/blog/2017/practice-makes-perfect-using-practice-standards-increase-classroom-engagement/>
- ◎ <http://map.mathshell.org/stds.php?standardid=1159>
- ◎ <http://commoncoretools.me/wp-content/uploads/2014/02/Elaborations.pdf>
- ◎ <https://achievethecore.org>